AP Calculus AB Summer Assignment Name________________________________

DUE: August 8, 2022

This assignment is for students who have completed Advanced Math Honors and are taking AP Calculus AB in the 2022-2023 school year.

Did you read the instructions? ________

What math are you taking in the 2022-2023 school year? ______________________

The expectation of the Math Department at Archbishop Hannan High School is that its students become Tenacious Problem Solvers! Thus, as you work on these problems be sure and document your strategies, your mathematical explanations, any drawings, tables or graphs that you use, and the best, complete answer you can find. We hope that you are challenged by these problems and enjoy them. We look forward to the discussion of these problems that we will have in the first weeks of school. Come prepared to defend your solution!

1. After being dropped from the top of a tall building, the height of an object is described by $y = 400 - 16t^2$, where y is measured in feet and t is measured in seconds.

(a) How many seconds did it take for the object to reach the ground?

(b) How high is the projectile when $t = 2$?

2. Find a function f for which $f(x+3)$ is *not* equivalent to $f(x)+f(3)$. Then find an f for which $f(x+3)$ *is* equivalent to $f(x)+f(3)$.

3. The x-intercepts of $y = f(x)$ are –1, 3, and 6. Find the x-intercepts of the following functions. Compare the appearance of each graph to the appearance of the graph $y = f(x)$.

(a) $y = f(2x)$ (b) $y = 2f(x)$ (c) $y = f(x+2)$ (d) $y = f(mx)$

4. Garbanzo bean cans usually hold 4000 cc (4 liters). It seems likely that the manufacturers of these cans have chosen the dimensions so that the material required to enclose 4000 cc is as small as possible. Let's find out what the optimal dimensions are.

(a) Find an example of a right circular cylinder whose volume is 4000. Calculate the total surface area of your cylinder, in square cm.

(b) Express the height and surface area of such a cylinder as a function of its radius r.

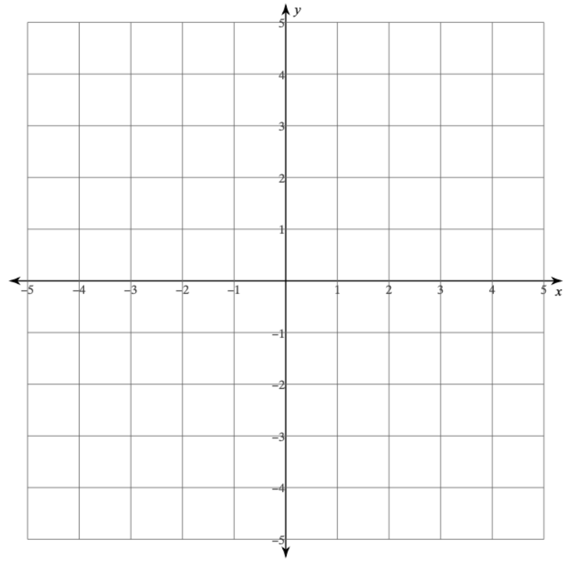
(c) Find the value of r that gives a cylinder of volume 4000 the smallest total surface area that it can have and calculate the resulting height.

5. It is true that $(5/6)^n < 0.0001$ for all sufficiently large values of n. How large is "sufficiently large"?

6. Given a function f, each solution to the equation $f(x)=0$ is called a *zero* of f. Without using a calculator, find the zeros of the following functions. Show your work to receive credit.

(a) $s(x)=\sin(3x)$ (b) $L(x)=\log_5(x-3)$ (c) $r(x)=\sqrt{2x+5}$ (d) $p(x)=x^3-4x$

7. Sketch the graph of $f(x)=\dfrac{x^2}{x-1}$. Identify all asymptotic behavior. What are the domain and range of f?



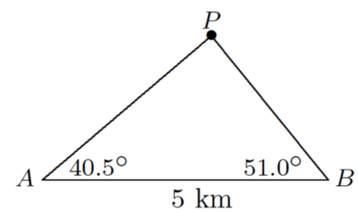
8. Rewrite the expression $\dfrac{3^{a+h}-3^a}{h}$ so that 3^a appears as a factor. Then use a calculator to evaluate $\lim\limits_{h\to 0}\dfrac{3^h-1}{h}$.

9. A driver was overheard saying "My trip to New York City was made at 80 kilometers per hour". Do you think the driver was referring to an *instantaneous* speed or an *average* speed? What is the difference between the two ideas?

10. You are driving on a straight highway on which the speed limit is 55mph. At 8:05am a police car clocks your velocity at 50mph and at 8:10am a second police car 5 miles down the road clocks your velocity at 55mph. Can the police charge you with speeding? Explain your reasoning.

11. Starting at the same spot on a circular track that is 80 meters in diameter, Hillary and Eugene run in opposite directions, at 300 meters per minute and 240 meters per minute, respectively. They run for 50 minutes. What distance separates Hillary and Eugene when they finish? There is more than one way to interpret the word *distance* in this question.

12. Two observers who are 5 km apart simultaneously sight a small airplane flying between them. One observer measures a 50.0° inclination angle, while the other observer measures a 40.5° inclination angle as shown in the diagram. At what altitude is the airplane flying?



13. Explain why the equation $\tan\theta = -2$ has solutions, but the equation $\sin\theta = -2$ does not.

14. Tickets for a concert were sold in three categories: adult, child and senior citizen. For each type, the number of tickets sold for the three performances is shown in the matrix. The box office receipts were \$2715 for Friday, \$2613 for Saturday, and \$2412 for Sunday. Find the cost of each type of ticket.

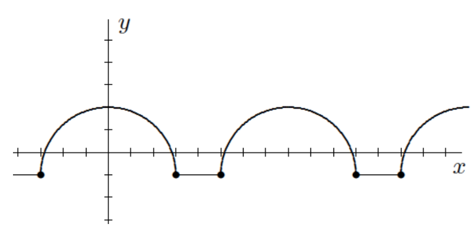
$$\begin{array}{c c} & \begin{array}{ccc} Child & Adult & Senior \end{array} \\ \begin{array}{c} Fri \\ Sat \\ Sun \end{array} & \begin{bmatrix} 35 & 120 & 15 \\ 22 & 124 & 12 \\ 58 & 96 & 6 \end{bmatrix} \end{array}$$

15. Solve for x: $4^{2016} - 4^{2015} - 4^{2014} + 4^{2013} = 90\left(2^x\right)$. Show your work to receive credit.

16. Solve each of the following equations by hand and explain why all three have the same solution.

(a) $8^x = 32$ (b) $27^x = 243$ (c) $1000^x = 100000$

17. What is half of 2^{40}? What is one third of 3^{18}? Explain your reasoning.

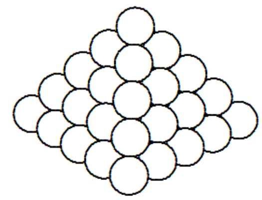


18. The figure at the right shows the graph $y = f(x)$ of a periodic function. The graph, whose period is 8, is built from segments and circular arcs. Notice the values $f(3) = -1$ and $f(5) = -1$. Calculate the following and explain your reasoning.

(a) $f(320)$ (b) $f(323)$ (c) $f(558)$

19. When the *binomial* power $(h+t)^6$ is expanded, what is the coefficient of the h^2t^4 term?

20. A grocer has 1015 spherical grapefruit, which are to be stacked in a square pyramid – one in the top layer, 4 in the next layer, etc. How many layers will the complete pyramid have?



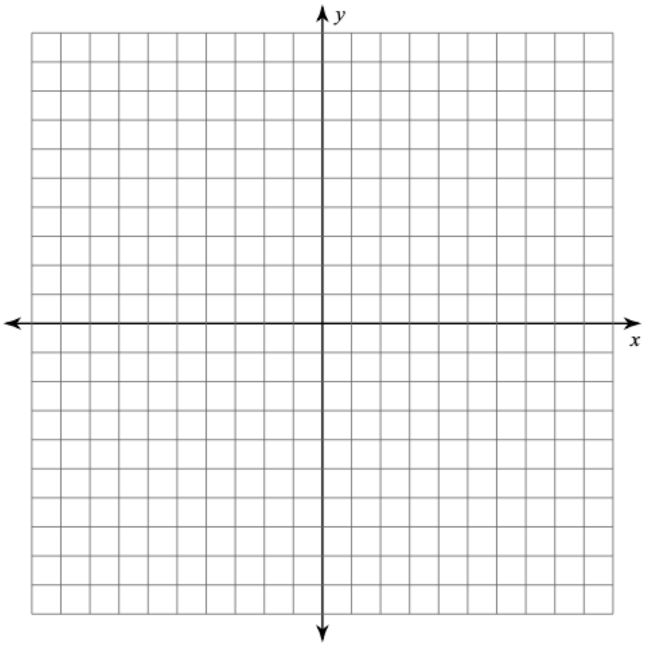
Essential Skills

The following problems represent the essential skills you need to be successful in AP Calculus AB.

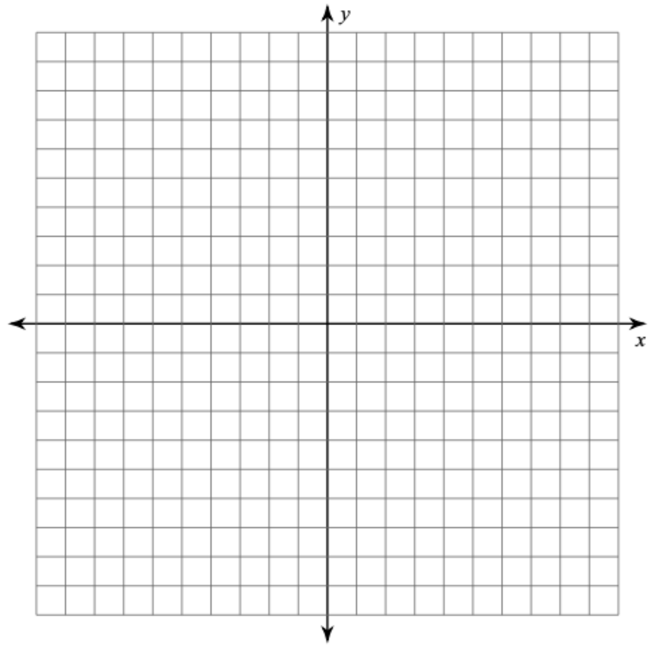
1. Divide. Write you answer in fraction form. $(2x^4 - 15x^2 + 18x + 22) \div (x+3)$

2. Graph the function. List holes, asymptotes, and zeroes.

a. $f\left(x\right)=\dfrac{2x^2+6x}{x^2+4x}$

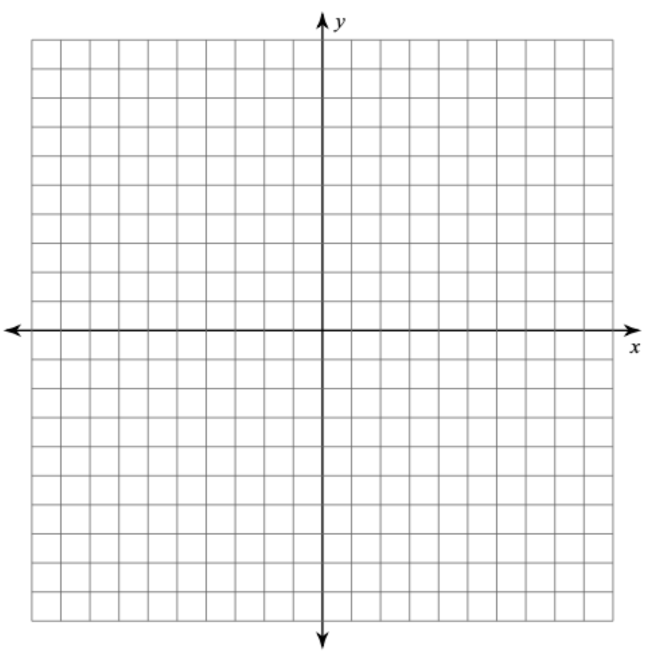


b. $f\left(x\right)=\dfrac{x^3-16x}{4x^2+8x-32}$

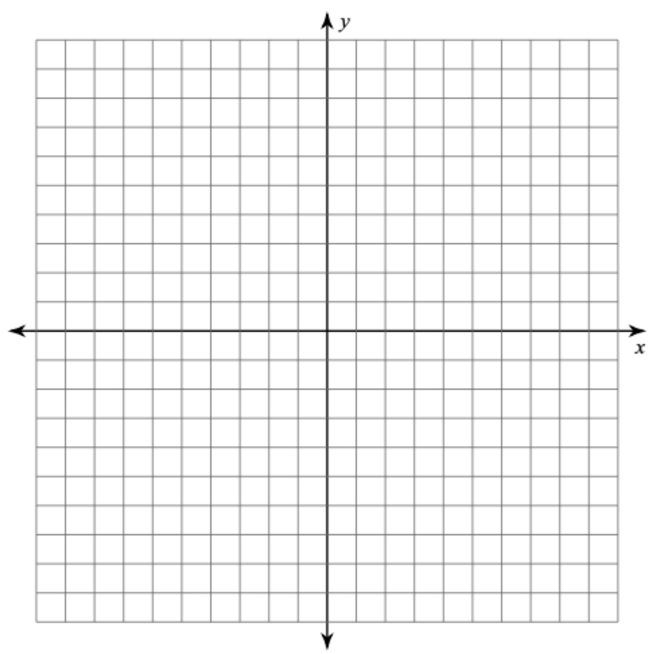


3. Graph the conic. List any important points.

a. $\left(x+3\right)^2+\left(y+1\right)^2=4$



b. $\dfrac{\left(x-3\right)^2}{36}+\dfrac{\left(y+1\right)^2}{4}=1$



4. Solve the equation. Give the exact answer. NO CALCULATOR.

a. $16 \bullet 4^{-x} = 64$

b. $\dfrac{81^{2-2x}}{27} = 9$

c. $\log_6(6-5r) = \log_6(-3r+10)$

d. $\log_5(-6x+3) = \log_5(x^2-13)$

e. $-1 + \log_{11}(-3m-6) = 1$

f. $\log_7(x-6) - \log_7 x = 2$

5. Solve the equation. Round your answer to 3 decimal places.

a. $3e^{9x} = 23.4$

b. $3 \cdot 6^{-5p} + 5 = 67$

6. Expand the logarithm.

a. $\log_8 \sqrt[3]{uvw}$

b. $\log_3 \dfrac{9x^5}{y^7}$

7. Condense the logarithm.

a. $5\log_8 u + \dfrac{1}{2}\log_8 v$

b. $3\log_9 x - 2\log_9(y+2) - \log_9 z$

8. Evaluate. NO CALCULATOR.

a. $\log_{36} 216$

b. $\log_{27} 9$

c. $\log_{\frac{1}{2}} \frac{1}{16}$

d. $\sin\left(-\frac{\pi}{3}\right)$

e. $\cot(0)$

f. $\cos\left(\frac{13\pi}{4}\right)$

g. $\sec\left(\frac{5\pi}{6}\right)$

h. $\tan\left(-\frac{\pi}{2}\right)$

i. $\sin^{-1}(2)$

j. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

k. $\tan^{-1}(1)$

l. $\csc\left(\frac{11\pi}{3}\right)$

9. Evaluate.

a. $\sum_{n=1}^{25} 3n^2 + 6$

b. $\sum_{n=1}^{\infty} 2 \cdot 3^{n-1}$

c. $\sum_{n=10}^{25} 7n - 5$